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A Principled Approach Towards Symbolic Geometric Constraint Satisfaction

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Abstract

An important problem in geometric reasoning is to find the configuration of a collection of geometric bodies so as to satisfy a set of given constraints. Recently, it has been suggested that this problem can be solved efficiently by *symbolically reasoning about geometry*. This approach, called *degrees of freedom analysis*, employs a set of specialized routines called *plan fragments* that specify how to change the configuration of a set of bodies to satisfy a new constraint while preserving existing constraints. A potential drawback, which limits the scalability of this approach, is concerned with the difficulty of writing plan fragments. In this paper we address this limitation by showing how these plan fragments can be automatically synthesized using first principles about geometric bodies, actions, and topology.

1. Introduction

An important problem in geometric reasoning is the following: given a collection of geometric bodies, called *geoms*, and a set of constraints between them, find a *configuration* – i.e., position, orientation, and dimension – of the geoms that satisfies all the constraints. Solving this problem is an integral task for many applications like constraint-based sketching and design, geometric modeling for computer-aided design, kinematics analysis of robots and other mechanisms (Hartenberg & Denavit, 1964), and describing mechanical assemblies.

General purpose constraint satisfaction techniques are not well suited for solving constraint problems involving complicated geometry. Such techniques represent geoms and constraints as algebraic equations, whose real solutions yield the numerical values describing the desired configuration of the geoms. Such equation sets are highly non-linear and highly coupled and in the general case require iterative numerical solutions techniques. Iterative numerical techniques are not particularly efficient and can have problems with stability and robustness (Press, Flannery, Teukolsky & Vetterling, 1986). For many tasks (e.g., simulation and optimization of mechanical devices) the same equations are solved repeatedly which makes a compiled solution desirable. In theory, symbolic manipulation of equations can often yield a non-iterative, closed form solution. Once found, such a closed-form solution can be executed very efficiently.

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