SC-FDE

Single-Carrier Block Transmission With Frequency-Domain Equalisation

Professor Sheng Chen
Electronics and Computer Science
University of Southampton
Southampton SO17 1BJ, UK
E-mail: sqc@ecs.soton.ac.uk
Motivations

• For 4G and B4G **high-speed broadband applications**, data rate of tens Mbps or higher over wireless channel of typical delay spread in microseconds
  
  – ISI spanning tens or even hundreds of symbols
  – Nightmare for time-domain equalisation: impractically long equaliser, excessively slow convergence ⇒ poor performance

• **Orthogonal frequency division multiplexing**, a multi-carrier technique, offers a viable low-complexity high-performance solution for ISI mitigation
  
  – High peak-to-average power ratio, intolerance to amplifier nonlinearity, and high sensitivity to carrier frequency offsets

• Alternative solution for ISI mitigation is **single-carrier** modulation with **frequency-domain equalisation**
  
  – Similar low-complexity and performance, but avoiding OFDM’s drawbacks
  – Not as flexible as OFDM in managing bandwidth and energy resources
Quick Comparison

Symbol mapping $\rightarrow$ S/P $\rightarrow$ Cyclic prefix insertion $\rightarrow$ P/S $\rightarrow$ Wireless channel

P/S $\rightarrow$ detection $\rightarrow$ IFFT $\rightarrow$ Frequency domain equalisation $\rightarrow$ FFT $\rightarrow$ S/P $\rightarrow$ prefix removal

Symbol mapping $\rightarrow$ S/P $\rightarrow$ IFFT $\rightarrow$ Cyclic prefix insertion $\rightarrow$ P/S $\rightarrow$ Wireless channel

P/S $\rightarrow$ equalisation and detection $\rightarrow$ FFT $\rightarrow$ S/P $\rightarrow$ prefix removal

SC-FDE

OFDM

Both transceivers have similar implementation complexity, but SC-FDE transmitter is simpler and hence better for uplink handset.
Block Transmission

• Data symbols \( \{ s_n \} \) are transmitted in blocks of \( N \) symbols with cyclic prefix of length \( M \) \( \Rightarrow \) block length of \( N + M \)
  
  - \( M \) is chosen to be larger than channel impulse response length

• Cyclic prefix insertion

  \[
  \begin{array}{c|c|c}
  \text{last } M \\
  \text{symbols copied} & \text{block of } N \text{data symbols} & \text{last } M \\
  \text{symbols} & \text{symbols} & \text{copying}
  \end{array}
  \]

  - Last \( M \) symbols may be training symbols, e.g. known PN sequence

• Spot “difference”

  - SC-FDE: data symbols are “time-domain” quantity and transmitted directly
  - OFDM: data symbols are “frequency-domain” quantity (and are IDFT into “time-domain” for transmission)
OFDM

- **Cyclic prefix** at beginning of each block has two main functions
  - Prevent contamination of a block by ISI from previous block, by simply dropping first $M$ time-domain samples of received block of length $N + M$
  - Make received block **periodic with period** $N$, essential for DFT to lead single-tap equalisation in frequency domain
- Received block has **cyclicity property**
  \[
  r_m = \sum_{k=0}^{N_h-1} h_k s_{m-(k \mod N)}, \quad 0 \leq m \leq N - 1
  \]
  where $N_h \leq M$ is length of CIR, and $h_k, 0 \leq k \leq N_h - 1$, CIR taps
- Process received block $\{r_m\}_{m=0}^{N-1}$ by DFT:
  \[
  R_l = \sum_{m=0}^{N-1} r_m e^{-j \frac{2\pi lm}{N}} = H_l \cdot S_l + V_l, \quad 0 \leq l \leq N - 1
  \]
  CFR $\{H_l\}_{l=0}^{N-1}$ is $N$-point DFT of CIR $\{h_k\}_{k=0}^{N_h-1}$, $V_l$ is noise term
- One-tap equalisation
  \[
  Y_l = W_l \cdot R_l, \quad 0 \leq l \leq N - 1
  \]
  $\{Y_l\}_{l=0}^{N-1}$ provides sufficient statistics for **detection** of transmitted data symbols $\{S_l\}_{l=0}^{N-1}$
Cyclic Prefix and Cyclicity

- Transmitted time-domain OFDM signal: \( s_{N+M} = [s_{N-1} s_{N-2} \cdots s_{N-M} \cdots s_1 s_0 | s_{-1} s_{-2} \cdots s_{-M}]^T \)
  
  - \( N \) data symbols \( s_N = [s_{N-1} \cdots s_1 s_0]^T \), and
  
  - \( M \)-length cyclic prefix \( [s_{-1} s_{-2} \cdots s_{-M}]^T = [s_{N-1} s_{N-2} \cdots s_{N-M}]^T \)
  
  - \( M \geq \text{CIR length} \), and for convenience, let CIR be: \([h_0 h_1 \cdots h_M]^T\)

- Received block of length \( N + M \): \( r_{N+M} = [r_{N-1} \cdots r_1 r_0 | r_{-1} r_{-2} \cdots r_{-M}]^T \)
  
  - Dropping \( [r_{-1} r_{-2} \cdots r_{-M}]^T \) removes ISI from previous block
  
  - \( N \)-length received block \( r_N = [r_{N-1} \cdots r_1 r_0]^T \) has cyclicity property

- Linear convolution \( r_N = H_s s_{N+M} \), \( H_s : N \times (N + M) \)

\[
\begin{align*}
  r_{N-1} &= h_0 s_{N-1} + h_1 s_{N-2} + \cdots + h_M s_{N-M+1} \\
  \vdots \\
  r_M &= h_0 s_M + h_1 s_{M-1} + \cdots + h_M s_0 \\
  r_{M-1} &= h_0 s_{M-1} + h_1 s_{M-2} + \cdots + h_{M-1} s_0 + s_M s_{-1} \\
  \vdots \\
  r_1 &= h_0 s_1 + h_1 s_0 + h_2 s_{-1} + \cdots + h_{M} s_{-M+1} \\
  r_0 &= h_0 s_0 + s_1 s_{-1} + \cdots + h_M s_{-M}
\end{align*}
\]
Cyclicity (continue)

\[
\begin{bmatrix}
    r_{N-1} \\
    \vdots \\
    r_M \\
    r_{M-1} \\
    \vdots \\
    r_1 \\
    r_0
\end{bmatrix}
= 
\begin{bmatrix}
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M
\end{bmatrix}
\begin{bmatrix}
    s_{N-1} \\
    s_{N-2} \\
    \vdots \\
    s_{N-M} \\
    \vdots \\
    s_0 \\
    s_{-1} \\
    s_{-2} \\
    \vdots \\
    s_{-M}
\end{bmatrix}
\]

- **Circular convolution** \( r_N = H_C s_N \), \( H_C : N \times N \)
Block Processing in SC-FDE

- **Cyclic prefix** at beginning of each block has two main functions
  - Prevent contamination of a block by intersymbol interference from previous block, by simply dropping first $M$ samples of received block of length $N + M$
  - Make received block *periodic with period* $N$, essential for DFT to lead single-tap equalisation in frequency domain

- Received block has *cyclicity property*

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    r_m = \sum_{k=0}^{N_h-1} h_k s_{m-(k \mod N)}, \quad 0 \leq m \leq N - 1
\]

where $N_h \leq M$ is length of CIR, and $h_k$, $0 \leq k \leq N_h - 1$, CIR taps

- Process received block $\{r_m\}_{m=0}^{N-1}$ by DFT:

\[
    R_l = \sum_{m=0}^{N-1} r_m e^{-j\frac{2\pi lm}{N}} = H_l \cdot S_l + V_l, \quad 0 \leq l \leq N - 1
\]

CFR $\{H_l\}_{l=0}^{N-1}$ is $N$-point DFT of CIR $\{h_k\}_{k=0}^{N_h-1}$, $V_l$ is noise term
One-Tap Equalisation

- In **frequency-domain**, equalisation can be achieved by **one-tap** linear equaliser

\[ Y_l = W_l \cdot R_l, \quad 0 \leq l \leq N - 1 \]

- **Zero-forcing**:

\[ W_l = \frac{H_l^*}{|H_l|^2}, \quad 0 \leq l \leq N - 1 \]

- **Minimum mean square error**:

\[ W_l = \frac{H_l^*}{|H_l|^2 + \frac{\sigma_v^2}{\sigma_s^2}}, \quad 0 \leq l \leq N - 1 \]

where \( \sigma_v^2 \) is noise power and \( \sigma_s^2 = E[|s_n|^2] \) signal power

- SC-FDE with decision feedback equaliser

- \( \{y_n\}_{n=0}^{N-1} \), IDFT of \( \{Y_l\}_{l=0}^{N-1} \)

\[ y_n = \frac{1}{N} \sum_{l=0}^{N-1} Y_l e^{j \frac{2\pi ln}{N}}, \quad 0 \leq n \leq N - 1 \]

provides sufficient statistics for **detection** of transmitted data symbols \( \{s_n\}_{n=0}^{N-1} \)
Quick Comparison Again

- OFDM and SC-FDE have similar low-complexity and performance
  - OFDM has high peak to average power ratio, and sensitive to carrier phase noise
  - SC-FDE requires much tighter pulse shaping with associated higher complexity
- For single-carrier systems with wideband signals, pulse shaping must be very tight, say $\gamma \leq 0.1$
  - Channel bandwidth $B_p$ is very large, 0.1 of which may be significant compared with guard band
- For OFDM, subcarrier spacing is small in comparison to guard band
  - In fact, often no pulse shaping is required for multi-carrier OFDM systems
FD Decision Feedback Equalizer

- SC-FDE and OFDM are basically linear equalisers
  - Suffer from usual noise enhancement
  - Performance degradation over deep frequency-selective fading channels

- Frequency-domain decision feedback equaliser: an iterative structure

\[
\{r_m\} \xrightarrow{\text{DFT}} \{R_l\} \xrightarrow{W_l} \{Y_l\} \xrightarrow{\text{IDFT}} \{s_m\} \xrightarrow{\hat{s}_m} \\
\{B_l\} \xrightarrow{\text{DFT}} \{S_l\} \xrightarrow{\hat{S}_l} \\
\]

- At \(k\)th iteration, feedforward equaliser \(\{W_l^{(k)}\}\), and feedback equaliser \(\{B_l^{(k)}\}\)
- First iteration is usual FD linear equaliser
- Many ways of updating \(\{W_l^{(k)}\}\) and \(\{B_l^{(k)}\}\)

OFDMA / SC-FDMA

- Multi-carrier system to support multi users: orthogonal frequency division multiple access OFDMA
  - $N$ subcarriers to support $K$ users
  - Carrier assignment scheme CAS assigns $N/K$ subcarriers to each user

```
User 1  ○○○○●○○○●○○○●○○○●○○○●○○○●○○○●○○○●
User 2  ○●○○○●○○○●○○○●○○○●○○○●○○○●○○○●
User 3  ○○●○●○○●○○○●○○○●○○○●○○○●○○○●
User 4  ●○○○○●○○○●○○○●○○○●○○○●○○○●
```

4 users 16 subcarriers

- Each user assigns its data symbols to subcarriers it occupies, and assigns zero to unoccupied subcarriers

- Single-carrier system to support multi users: frequency division multiple access SC-FDMA

```
user 3  user 2  user 4  user 1
```

Total bandwidth

- For MC systems, such as OFDMA, flexible for resource allocation, such as power allocation to subcarriers
Summary

• Single-carrier block transmission with frequency-domain equalisation
  – Alternative to OFDM, similarity with OFDM
  – Advantages/disadvantage comparison with OFDM

• Cyclic prefix and Cyclicity: remove inter-block interference and make received block periodic
  – Frequency-domain one-tap equalisation

• Frequency-domain decision feedback equalisation

• Support multi users: OFDMA and SC-FDMA