Single-Carrier Block Transmission With Frequency-Domain Equalisation

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Motivations

• For 4G and B4G **high-speed broadband applications**, data rate of tens Mbps over wireless channel of typical delay spread in microseconds
  - ISI spanning tens or even hundreds of symbols
  - Nightmare for time-domain equalisation: impractically long equaliser, excessively slow convergence ⇒ poor performance

• **Orthogonal frequency division multiplexing**, a multi-carrier technique, offers a viable low-complexity high-performance solution for ISI mitigation
  - High peak-to-average power ratio, intolerance to amplifier nonlinearity, and high sensitivity to carrier frequency offsets

• Alternative solution for ISI mitigation is **single-carrier** modulation with **frequency-domain equalisation**
  - Similar low-complexity and performance, but avoiding OFDM’s drawbacks
  - Not as flexible as OFDM in managing bandwidth and energy resources
Quick Comparison

Both transceivers have similar implementation complexity, but SC-FDE transmitter is simpler and hence better for uplink handset.

SC-FDE

OFDM
Block Transmission

- Data symbols \( \{s_n\} \) are transmitted in blocks of \( N \) symbols with cyclic prefix of length \( M \) ⇒ block length of \( N + M \)
  - \( M \) is chosen to be larger than channel impulse response length

- Cyclic prefix insertion

- Last \( M \) symbols may be training symbols, e.g. known PN sequence

- Spot "difference"
  - SC-FDE: data symbols are "time-domain" quantity and transmitted directly
  - OFDM: data symbols are "frequency-domain" quantity (and are IDFT into "time-domain" for transmission)
OFDM

• **Cyclic prefix** at beginning of each block has two main functions
  – Prevent contamination of a block by ISI from previous block, by simply dropping first $M$ time-domain samples of received block of length $N + M$
  – Make received block **periodic with period** $N$, essential for DFT to lead single-tap equalisation in frequency domain

• Received block has **cyclicity property**

$$r_m = \sum_{k=0}^{N_h-1} h_k s_{m-(k \mod N)}, \quad 0 \leq m \leq N - 1$$

where $N_h \leq M$ is length of CIR, and $h_k$, $0 \leq k \leq N_h - 1$, CIR taps

• Process received block $\{r_m\}_{m=0}^{N-1}$ by DFT:

$$R_l = \sum_{m=0}^{N-1} r_m e^{-j\frac{2\pi lm}{N}} = H_l \cdot S_l + V_l, \quad 0 \leq l \leq N - 1$$

CFR $\{H_l\}_{l=0}^{N-1}$ is $N$-point DFT of CIR $\{h_k\}_{k=0}^{N_h-1}$, $V_l$ is noise term

• One-tap equalisation

$$Y_l = W_l \cdot R_l, \quad 0 \leq l \leq N - 1$$

$\{Y_l\}_{l=0}^{N-1}$ provides sufficient statistics for **detection** of transmitted data symbols $\{S_l\}_{l=0}^{N-1}$
Cyclic Prefix and Cyclicity

- Transmitted time-domain OFDM signal: 
  \[ s_{N+M} = \begin{bmatrix} s_{N-1} & s_{N-2} & \cdots & s_{N-M} & \cdots & s_1 & s_0 \end{bmatrix}^T \]
  - \( N \) data symbols \( s_N = \begin{bmatrix} s_{N-1} & \cdots & s_1 & s_0 \end{bmatrix}^T \), and
  - \( M \)-length cyclic prefix \( \begin{bmatrix} s_{-1} & s_{-2} & \cdots & s_{-M} \end{bmatrix}^T = \begin{bmatrix} s_{N-1} & s_{N-2} & \cdots & s_{N-M} \end{bmatrix}^T \)
  - \( M \geq \) CIR length, and for convenience, let CIR be: \( \begin{bmatrix} h_0 & h_1 & \cdots & h_M \end{bmatrix}^T \)

- Received block of length \( N + M \): 
  \[ r_{N+M} = \begin{bmatrix} r_{N-1} & \cdots & r_1 & r_0 & r_{-1} & r_{-2} & \cdots & r_{-M} \end{bmatrix}^T \]
  - Dropping \( r_{-1} r_{-2} \cdots r_{-M} \) removes ISI from previous block
  - \( N \)-length received block \( r_N = \begin{bmatrix} r_{N-1} & \cdots & r_1 & r_0 \end{bmatrix}^T \) has cyclicity property

- Linear convolution \( r_N = H_L s_{N+M} \), \( H_L : N \times (N + M) \)

\[
\begin{align*}
    r_{N-1} &= h_0 s_{N-1} + h_1 s_{N-2} + \cdots + h_M s_{N-M+1} \\
    &\vdots \\
    r_M &= h_0 s_{M} + h_1 s_{M-1} + \cdots + h_M s_0 \\
    r_{M-1} &= h_0 s_{M-1} + h_1 s_{M-2} + \cdots + h_{M-1} s_0 + s_M s_{-1} \\
    &\vdots \\
    r_1 &= h_0 s_{1} + h_1 s_{0} + h_2 s_{-1} + \cdots + h_M s_{-M+1} \\
    r_0 &= h_0 s_{0} + s_1 s_{-1} + \cdots + h_M s_{-M}
\end{align*}
\]
Cyclicity (continue)

\[
\begin{bmatrix}
    r_{N-1} \\
    \vdots \\
    r_M \\
    r_{M-1} \\
    \vdots \\
    r_1 \\
    r_0
\end{bmatrix}
= \begin{bmatrix}
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
\end{bmatrix}
\begin{bmatrix}
    s_{N-1} \\
    s_{N-2} \\
    \vdots \\
    s_{N-M} \\
    \vdots \\
    s_0 \\
    s_{-1} \\
    s_{-2} \\
    \vdots \\
    s_{-M}
\end{bmatrix}
\]

- **Circular convolution** \( r_N = H_C s_N \), \( H_C : N \times N \)

\[
\begin{bmatrix}
    r_{N-1} \\
    \vdots \\
    r_M \\
    r_{M-1} \\
    \vdots \\
    r_1 \\
    r_0
\end{bmatrix}
= \begin{bmatrix}
    h_M \\
    \vdots \\
    \vdots \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
\end{bmatrix}
\begin{bmatrix}
    s_{N-1} \\
    s_{N-2} \\
    \vdots \\
    s_{N-M} \\
    \vdots \\
    s_0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
    h_M \\
    \vdots \\
    \vdots \\
    h_0 & h_1 & \cdots & h_{M-1} & h_M \\
\end{bmatrix}
\begin{bmatrix}
    s_{N-1} \\
    s_{N-2} \\
    \vdots \\
    s_{N-M} \\
    \vdots \\
    s_0 \\
\end{bmatrix}
\]
Block Processing in SC-FDE

• **Cyclic prefix** at beginning of each block has two main functions
  
  – Prevent contamination of a block by intersymbol interference from previous block, by simply dropping first $M$ samples of received block of length $N + M$
  
  – Make received block **periodic with period** $N$, essential for DFT to lead single-tap equalisation in frequency domain

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One-Tap Equalisation

- In **frequency-domain**, equalisation can be achieved by **one-tap** linear equaliser
  \[ Y_l = W_l \cdot R_l, \quad 0 \leq l \leq N - 1 \]

  - **Zero-forcing**:
    \[ W_l = \frac{H_l^*}{|H_l|^2}, \quad 0 \leq l \leq N - 1 \]

  - **Minimum mean square error**:
    \[ W_l = \frac{H_l^*}{|H_l|^2 + \frac{\sigma_v^2}{\sigma_s^2}}, \quad 0 \leq l \leq N - 1 \]

  where \( \sigma_v^2 \) is noise power and \( \sigma_s^2 = E[|s_n|^2] \) signal power

- SC-FDE with decision feedback equaliser

- \( \{y_n\}_{n=0}^{N-1} \), IDFT of \( \{Y_l\}_{l=0}^{N-1} \)
  \[ y_n = \frac{1}{N} \sum_{l=0}^{N-1} Y_l e^{\frac{j2\pi ln}{N}}, \quad 0 \leq n \leq N - 1 \]

  provides sufficient statistics for **detection** of transmitted data symbols \( \{s_n\}_{n=0}^{N-1} \)
OFDMA / SC-FDMA

- Multi-carrier system to support multi users: orthogonal frequency division multiple access **OFDMA**
  - $N$ subcarriers to support $K$ users
  - Carrier assignment scheme **CAS** assigns $N/K$ subcarriers to each user

<table>
<thead>
<tr>
<th>User 1</th>
<th>User 2</th>
<th>User 3</th>
<th>User 4</th>
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<td>○○○○○</td>
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 4 users 16 subcarriers

- Each user assigns its data symbols to subcarriers it occupies, and assigns zero to unoccupied subcarriers

- Single-carrier system to support multi users: frequency division multiple access **SC-FDMA**

- For MC systems, such as OFDMA, flexible for resource allocation, such as **power allocation** to subcarriers
References


