Blind FIR Equalisation for High-Order QAM Signalling

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Overview

Low-complexity affordable blind equalisation for high-order QAM

- Constant modulus algorithm
- Concurrent CMA and decision-directed scheme (De Castro et al, ICC'2001)
- Multi-stage blind clustering or bootstrap maximum a posteriori probability scheme (Chen et al, ICC'1993)

Comparative study

Blind Equalisation

- Baseband channel model

\[ r(k) = \sum_{i=0}^{n-1} a_i s(k-i) + e(k) \]

- \( a_i = a_i R + j a_i I \): channel taps; \( s(k) = s_R(k) + j s_I(k) \in M\)-QAM set:

\[ S = \{ s_{il} = (2i - Q - 1) + j(2l - Q - 1), \quad 1 \leq i, l \leq Q \} \]

- With \( Q = \sqrt{M} = 2^L \) and \( L \) an integer; \( e(k) = e_R(k) + j e_I(k) \): Gaussian white noise with \( E[v_R^2(k)] = E[v_I^2(k)] = \sigma_e^2 \)

- Equaliser

\[ y(k) = \sum_{i=0}^{m-1} w_i r(k-i) = w^T r(k) \]

- \( w(k) = [w_0 \; w_1 \cdots w_{m-1}]^T \): equaliser weight vector with \( w_i = w_i R + j w_i I \); \( r(k) = [r(k) \; r(k-1) \cdots r(k-m+1)]^T \): equaliser input vector

Concurrent CMA and DD Scheme

- De Castro et al (ICC'2001): \( w = w_c + w_d \)

- CMA equaliser \( w_c \) minimises

\[ J_{\text{CMA}}(w) = E \left[ (|y(k)|^2 - \Delta_2)^2 \right] \]

\[ e(k) = y(k) (\Delta_2 - |y(k)|^2) \]

\[ w_c(k+1) = w_c(k) + \mu_c e(k) r^*(k) \]

- DD equaliser \( w_d \) minimises

\[ J_{\text{DD}}(w) = \frac{1}{2} E \left[ |Q[y(k)] - y(k)|^2 \right] \]

with quantized equalizer output defined by

\[ Q[y(k)] = \arg \min_{s_{il} \in S} |y(k) - s_{il}|^2 \]
DD adaptation follows CMA adaptation:

\[ w_{a}(k + 1) = w_{a}(k) + \mu_{d}\delta(Q[y(k)] - Q[y(k)]) \delta(y(k)) \]

where \( Q[y(k)] \) and \( Q[y(k)] \) are equaliser hard decisions after and before CMA adaptation, and indicator function

\[ \delta(x) = \begin{cases} 1, & x = 0 + j0 \\ 0, & x \neq 0 + j0 \end{cases} \]

- \( w_{a} \) is updated only if equaliser hard decisions before and after CMA adaptation are the same
- Hard decision directed adaptation: if decision is wrong \( \Rightarrow \) error propagation
- If equaliser hard decisions before and after CMA adaptation are the same, decision is probably right one

Bootstrap MAP equaliser maximises log of the \( a \) posteriori PDF criterion

\[ J_{\text{MAP}}(w) = E[J_{\text{MAP}}(w, y(k))] \]

where

\[ J_{\text{MAP}}(w, y(k)) = \rho \log(p(w, y(k))) \]

using stochastic gradient algorithm

\[ w(k + 1) = w(k) + \mu \frac{\partial J_{\text{MAP}}(w(k), y(k))}{\partial w} \]

given stochastic gradient

\[ \frac{\partial J_{\text{MAP}}(w(k), y(k))}{\partial w} = \frac{\sum_{q=1}^{Q} \sum_{l=1}^{Q} \exp\left(\frac{-|y(k) - s_{ql}|^2}{2\rho}\right) (s_{ql} - y(k))}{\sum_{q=1}^{Q} \sum_{l=1}^{Q} \exp\left(\frac{-|y(k) - s_{ql}|^2}{2\rho}\right)} - r^{*}(k) \]

- \( L = \log_{2}(M)/2 \) stage procedure to achieve minimum complexity
Computational Complexity per Weight Update

<table>
<thead>
<tr>
<th>equaliser</th>
<th>multiplications</th>
<th>additions</th>
<th>(\exp(\cdot)) evaluations</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMA</td>
<td>(8 \times m + 6)</td>
<td>(8 \times m)</td>
<td>–</td>
</tr>
<tr>
<td>CMA+DD</td>
<td>(16 \times m + 8)</td>
<td>(20 \times m)</td>
<td>–</td>
</tr>
<tr>
<td>MAP</td>
<td>(8 \times m + 23)</td>
<td>(8 \times m + 19)</td>
<td>4</td>
</tr>
</tbody>
</table>

- \(m\) is equaliser order
- Four \(\exp(\cdot)\) evaluations implemented through look up table
- Tuning of bootstrap MAP equaliser more complicated, as it involves \(L\) stage switchings, each having a set of different algorithm parameters

16-QAM Example

22-tap channel, 23-tap equaliser, channel SNR = 25 dB

\[
\text{MD} = \sum_{d=0}^{n_{\text{code}}-1} \left| s_d - \hat{s}_{\text{code}} \right|/|s_{\text{code}}|
\]

with \(\{s_d\}_{d=0}^{n_{\text{code}}-1}\) combined impulse response of channel and equaliser

64-QAM Example

5-tap channel, 23-tap equaliser, channel SNR = 30 dB
256-QAM Example

22-tap channel, 23-tap equaliser, channel SNR = 80 dB

Conclusions

Two novel low-complexity blind equalisers: concurrent CMA and DD scheme and bootstrap MAP scheme

↑ Bootstrap MAP blind equaliser has simpler computational complexity per weight update, faster convergence rate, and marginally better steady-state equalisation performance

↓ Tuning of bootstrap MAP blind equaliser is more complicated, as it involves \( L \) stage switchings, each having a set of different algorithm parameters

○ Both offers significant improvement in equalisation performance over the very simple CMA