Improving snake performance via a dual active contour

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Abstract

A dual active contour can use two snakes to seek an energy minimum which lies between their initial positions. This relieves problems associated with initialisation. The contraction force is removed by the inclusion of local shape information within the snakes and the parameters required to control their evolution can be combined within a single regularisation parameter. An adaptive driving force is used to move the snakes which reduces the sensitivity to parameters. These developments are demonstrated to provide a performance superior to that of a conventional single snake approach.

Keywords: *Snakes, Active Contours, Dual Active Contour*

1 Introduction

Active contours (Kass et al., 1988), also termed snakes by the nature of their evolution, are a sophisticated approach to contour extraction and image interpretation. They challenge the widely held view that low-level vision tasks such as edge detection are 'bottom-up' processes; features are extracted from an image and higher level processes interpolate to find a suitable representation. The principal disadvantage with such an approach is its serial nature; errors generated at a low-level are passed on through the system without the possibility of correction. Active contours take an initial estimate based on higher-level knowledge and refine this estimate using an optimisation process. The principal advantage of the snake is that the image data, the initial estimate, desired contour properties and knowledge-based constraints are integrated into a single extraction process. The presence of an edge depends not only on the gradient at a specific point but also on the spatial distribution of the gradient. Snakes incorporate this global view of edge detection by assessing continuity and curvature combined with the local edge strength to determine an edge. The motivation for developing a dual active contour is to enhance the snake model by confronting its principal problems:

Initialisation The final extracted contour is highly dependent on the position and shape of the initial contour as a consequence of many local minima in the energy function. The initial contour must be placed near the required feature otherwise the contour can become obstructed by unwanted features.

Parameters The original technique gives no guidelines for determining the parameters. The values used are critical and must be chosen carefully to obtain meaningful results, due to scale variance and parameter inter-dependence.

Non-convex shapes By its formulation the original technique is poor at extracting non-convex shapes. A dual contour can provide a more balanced approach to contour extraction allowing both convex and non-convex shapes to be extracted.

Scale invariance The snake's internal energy is not scale invariant. This prevents the energy of the snake being used to assess the merit of the solution.

A dual contour is introduced to overcome the initialisation problems by approaching the desired feature from both sides. This contrasts with other methods which approach a feature from one side (Kass et al., 1988), (Cohen and Cohen, 1993) and hence have less ability to determine a global minimum. The new technique provides a more balanced approach and is better at extracting non-convex shapes. The internal energy of the contour is reformulated to be scale invariant, and allows a relative assessment to reject poor local minima. The new technique's implementation allows any additional local shape information (provided orientation can be estimated) to be integrated within the minimisation process. This shape model is similar to that of (Lai and Chin, 1994) in allowing local control over the contour's equilibrium. The problem of determining parameters is simplified by reducing the parameters to a single regularisation parameter which is consistent with the original paradigm.

2 A Local Shape Based Dual Active Contour

The dual technique uses an internal and external contour as the initial snakes (Gunn and Nixon, 1994). These contours have no tendency to expand or contract other than to attain a prior shape. A problem with the original technique was the continuity and curvature constraints resulted in a contraction force which produced a solution which is highly dependent on the internal parameters (Xu et al., 1994). In the dual contour the two contours are minimised according to a trade-off between the prior shape constraints and the image. When both contours become stationary, the contour with the highest energy is minimised with an additional force pushing the contour towards the other contour. This force is increased until the contour begins to move. If the energy begins to decrease, the driving force is removed and the contour is allowed to come into equilibrium. The process is then repeated until both contours have found the same equilibrium. The driving force gives the snake a hill climbing ability, which allows the snake to climb out of rejected local energy minima.

2.1 The Dual Contour

A single contour can be formulated to allow local shape information to be integrated within it. Two such contours are used within the dual technique. A single contour should have the following properties in order that it may be employed as one of the dual contours: its energy should be invariant to scale, rotation and translation, so that contours of different scales may be compared. It should also have an equilibrium when it is similar to an estimated contour. Consequently the contour has no preference to expand or contract, other than to acquire its natural shape.

To implement the contour properties described above we impose two constraints. Using a discrete contour given by, $\mathbf{v}_i = (x_i, y_i)$, where $i = 0 \dots N-1$ and N is the number of points. All subscript arithmetic is modulo N, and the contours are described in an anti-clockwise manner. One condition, can be used to promote an evenly

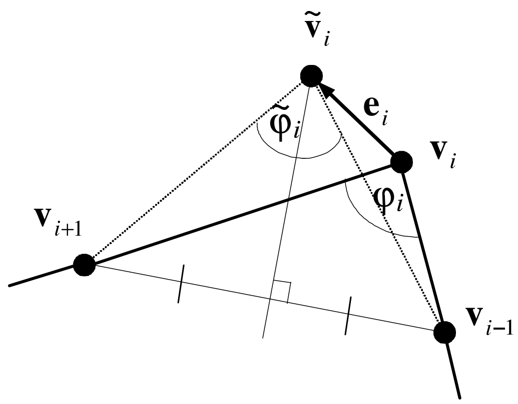


FIGURE 1: Local shape model

spaced set of contour points,

$$|\mathbf{v}_{i-1} - \mathbf{v}_i| = |\mathbf{v}_{i+1} - \mathbf{v}_i| \tag{1}$$

A second condition causes the contour to take a specified shape. In the case of a circle,

$$\varphi_i = \frac{N-2}{N}\pi \tag{2}$$

These conditions are applied by a force which pulls $\mathbf{v}_i$ towards its estimated position , $\tilde{\mathbf{v}}_i$. This force, $\mathbf{e}_i$, is calculated from the neighbouring points of $\mathbf{v}_i$ by,

$$\mathbf{e}_i = \tilde{\mathbf{v}}_i - \mathbf{v}_i = \frac{1}{2}(\mathbf{v}_{i-1} + \mathbf{v}_{i+1}) - \mathbf{v}_i + \theta_i \frac{1}{2}\mathbf{R}(\mathbf{v}_{i-1} - \mathbf{v}_{i+1}) \tag{3}$$

where $\mathbf{R}$ is a +90 degree rotation matrix. The internal energy of the contour at $\mathbf{v}_i$ is the energy associated with the force $\mathbf{e}_i$, normalised by the average space step, h, rendering the energy scale invariant.

$$E_{lshape}(\mathbf{v}_i) = \frac{1}{2}\left(\frac{|\mathbf{e}_i|}{h}\right)^2 \tag{4}$$

The internal energy term of Equation 4 is isotropic which is desirable whereas the original snake internal energy generally was not. The value of θ is related to the internal angle φ by, $\theta = \cot\left(\frac{\varphi}{2}\right)$. The circular case of Equation 4 gives

$$\theta = \cot\left(\frac{N-2}{2N}\pi\right) = \tan\left(\frac{\pi}{N}\right) \tag{5}$$

This will cause the contour to take the shape of a (discrete) circle. It can be verified that a circle is now a global solution to the internal energy. The new formulation allows any estimate of the shape to be included as an equilibrium state. A negative value of θ causes that part of the contour to become concave. However, it is now necessary to have an idea of the orientation of the shape in order to exploit this additional shape information, otherwise this may provide a worse estimate than the circular case. The complete contour energy equation is given by,

$$E_{snake}(\mathbf{v}) = \frac{1}{N}\sum_{i=0}^{N-1} \lambda E_{lshape}(\mathbf{v}_i) + (1-\lambda)E_{ext}(\mathbf{v}_i). \tag{6}$$

It is written in this form to emphasise the original paradigm of regularisation. The only parameter to be determined is the regularisation parameter, λ which has a value between 0 and 1. If $\lambda = 1$ the contour is completely regularised and depends only on internal forces for its solution. If $\lambda = 0$ then the contour is not regularised and no constraints on the contours shape or continuity are applied.

2.2 Implementation

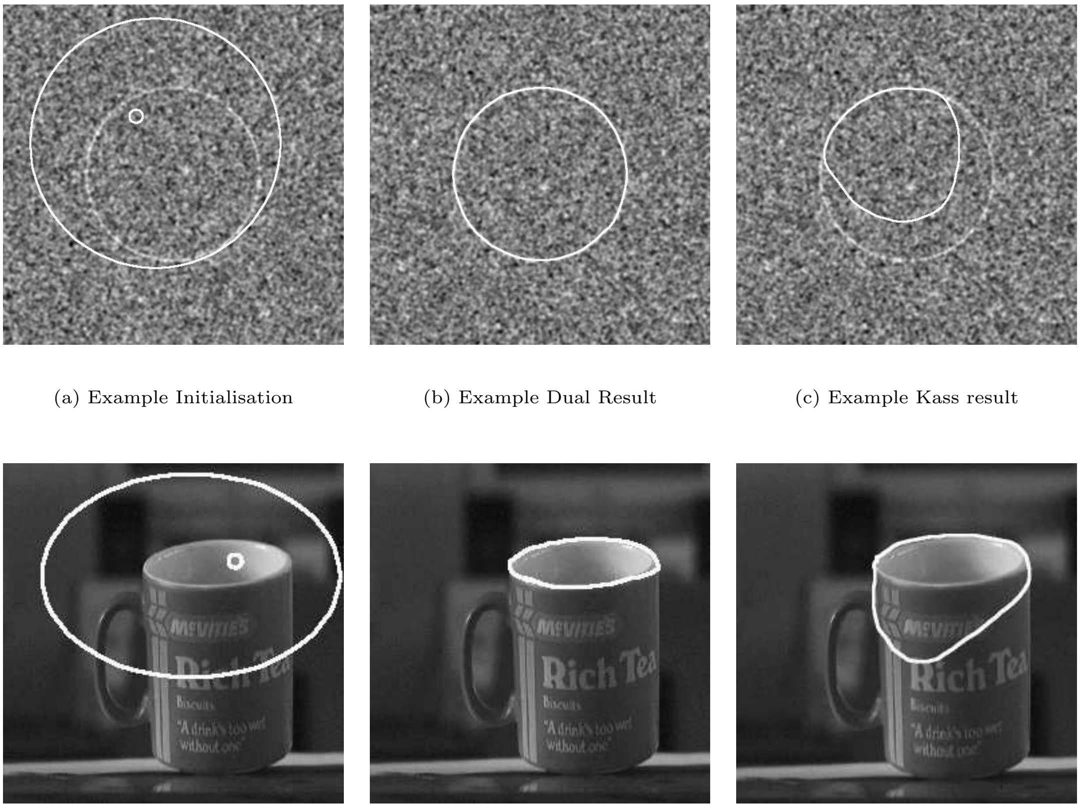
The minimisation process is implemented by the method of gradient descent. The external forces, $\mathbf{F}$, are derived from an edge-based image functional, $-|\nabla I(x,y)|$. The forces are globally normalised so that they lie within the interval $[-1, 1]$. Then the contour is evolved by,

$$\mathbf{v}_i^{\tau+1} = \mathbf{v}_i^{\tau} + \frac{1}{2}\left(\lambda\frac{\mathbf{e}_i}{h} + (1-\lambda)\mathbf{F}_i\right) \tag{7}$$

A continuous snake space is used as opposed to discrete space method such as (Williams and Shah, 1992) because a discrete space can introduce extra local minima within the internal energy function.

3 Results

To demonstrate the performance of the new technique it is compared with a conventional snake of (Kass et al., 1988). Results are shown for synthetic and real images. The image functionals used in the tests were simple edge based functionals. To evaluate the performance in the presence of noise a simulation test used a circle with added zero mean Gaussian noise using an image contrast of 1. The techniques are initialised with random initialisations, within and without the target circle, as appropriate. The outside contour for the dual snake was used as the initial contour for the original Kass snake. The regularisation parameter for the dual technique determines the trade-off between the prior shape information and the image data. A low value of λ causes the image forces to dominate the shape constraints and the contour adheres to the image data very well. However in the presence of noise we require a greater emphasis on the shape constraints and hence λ must be increased. Somewhere in between we require a trade-off between accurate representation to the data and accurate representation to the prior shape constraints. For the dual technique a value of $\lambda = 0.5$ was used. The determination of the parameters for the snake of (Kass et al., 1988) is a difficult task. In order to find a good set, many trial runs were performed and a manually optimised set of parameters was determined. Example



(a) Example Initialisation (b) Example Dual Result (c) Example Kass result

(d) Cup Initialisation (e) Dual Cup Result (f) Kass Cup Result

FIGURE 2: Example Results

results are shown in Figure 2 for the dual contour applied to the simulation data and to the image of a cup. The aim was to extract the circle in the simulation data and the cup rim. The example result confirms that the dual contour can identify the chosen target. In fact over runs of 10 initialisations for each image considered the dual technique performed better, working well in the simulation results up to a noise variance of 1.2, Figure 2(b). In contrast though the original Kass technique could and did select the target in some cases of the simulation data, but failed to for cases of poor initialisation and increased noise, Figure 2(c). Table 1 gives an average measure of the mean square error against the noise variance for a number of initialisations for each value of noise used. It can be seen that the performance of the dual technique is superior to the single snake approach in this test because the performance reduces less with increase in noise. However, if the snake parameters used here are now applied to other images with different degrees of minima the single snake may fail to be attracted by them or be attracted to insignificant local minima. Whereas the dual technique, by virtue of its adaptive driving force, is sensitive enough to extract different degrees of minima without the need to modify its parameters. The cup image, Figure 2(d), illustrates this case. As a consequence of the initialisation and the parameters used the original technique becomes snagged on complex image data, Figure 2(f); whereas the dual technique is able to avoid such minima Figure 2(e). In this paper we have developed a comprehensive dual contour technique

Noise STD	0.0	0.4	0.8	1.2	1.6	2.0
Original	0.31	0.44	0.70	22.6	23.8	40.0
Dual	0.12	0.19	0.29	2.82	19.5	38.4

Table 1: Results of MSE against Noise Variance

that overcomes the primary problems of sensitivity to initialisation and parameters associated with the original techniques. This sensitivity is a consequence of the original formulations in searching for a minimum. Typically there are many local minima and consequently the solution is highly dependent upon the parameters and initial contour. We search for a good minimum within the region specified by two initial contours. This is made possible by a reformulated internal energy which renders the snake energy scale invariant and provides a basis for assessing the merits of solutions. The original technique used the internal forces to provide a contraction of the contour, to move it towards features. This increased the sensitivity to the parameters so we replaced it with a new adaptive driving force which allows the contour to find minima, and to climb out of them if a better solution has been found by the other contour. Furthermore we argue that some shape information and orientation may be available, and if this is the case our technique is able to exploit it by integrating within the internal energy function. The results demonstrate that the technique provides good performance for simulated and real images with the same parameters.

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